correspond to the cases of the model surface temperature rise equal to or less than 10°C without correction, the triangles refer to cases of temperature increments in the range of 40° to 75°C without correction, and the squares refer to the same cases with correction.

It is evident from the previous discussion that, for temperature increments up to 75°C, the proposed correction circuit works very well. It is expected that it will also work satisfactorily for temperature increments up to 150°C or more.

References

[1] Skinner, G. T., "Analog network to convert surface temperature to heat flux," Cornell Aero. Lab. Rept. CAL-100 (February 1960); also ARS J. **30,** 569–570 (1960).

[2] Meyer, R. F., "A heat-flux-meter for use with thin film surface thermometers," National Research Council of Canada, Aeronautics Rept. LR-279 (April 1960).

[3] Hartunian, R. A. and Varwig, R. L., "On thin-film heat-transfer measurements in shock tubes and shock tunnels," Phys. Fluids **5,** 169–174 (1962).

[4] Walenta, Z. A., "Analogue networks for high heat-transfer rate measurements," Univ. of Toronto Institute for Aerospace Studies TN 84 (November 1964).

[5] Fay, J. A. and Riddell, F. R., "Theory of stagnation point heat transfer in dissociated air," J. Aeronaut. Sci. **25,** 73–85 (1958).

Velocity Defect Laws for Transpired Turbulent Boundary Layers

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In earlier articles[4, 5] the present authors have proposed a "velocity defect law" for the transpired turbulent boundary layer with $dp/dx = 0$:

$$(U_1 - U)/U_\tau^* = f(y/\delta) \qquad (1)$$

U_τ^* is the friction velocity based upon the shear stress at the inner edge of the outer flow.

Recently, Stevenson[7, 8] has proposed a "law of the wall"

$$\frac{2U_\tau}{v_0}\left\{\left(\frac{v_0 U}{U_\tau^2} + 1\right)^{1/2} - 1\right\} = \frac{1}{K}\ln\frac{yU_\tau}{\nu} + C \qquad (2)$$

and a "velocity defect law"

$$\frac{2U_\tau}{v_0}\left\{\left(\frac{v_0 U_1}{U_\tau^2} + 1\right)^{1/2} - \left(\frac{v_0 U}{U_\tau^2} + 1\right)^{1/2}\right\} = F\left(\frac{y}{\delta}\right) \qquad (3)$$

U_τ is the friction velocity based upon the wall shear stress. Stevenson showed that his experimental data, measured for axisymmetric flow with $dp/dx = 0$, $v_0 =$ const, followed Eqs. (2) and (3). He also found that the data of Mickley and Davis,[3] which was for a plane flow, fit his relations, although the plane-flow results differed moderately from the axisymmetric flow data. The question that immediately arises is: Are the two defect laws represented by Eqs. (1) and (3) compatible?

In Figs. 1 and 2, new data[2] obtained in the authors' laboratory with plane flow, $dp/dx = 0$, $U_1 = 25$ fps, and $v_0/U_1 = 0.00266/x^{0.2}$ (x in inches) are plotted in the form of Eq. (1) (Fig. 1) and Eq. (3) (Fig. 2). Universal curves are obtained for both correlations. The data of Ref. 3 for $dp/dx = 0$, $v_0 =$ const were also found to fall on the curves of Figs. 1 and 2. As a further check, Eqs. (2) and (3) were used in conjunction with the boundary-layer equations to calculate local shear stress as a function of y/δ for arbitrary positive values of v_0/U_1. The shear-stress profiles exhibited the expected broad maximum near $y/\delta = 0.1$, and $U_\tau^* = (\tau_{max}/\rho)^{1/2}$ was used to represent the friction velocity at the inner edge of the outer flow. A smooth curve through the data of Fig. 2 and representing Eq. (3) could then be converted to the form of Eq. (1) for arbitrary values of v_0/U_1. Up to the largest value of v_0/U_1 tested ($v_0/U_1 = 0.005$), a smooth curve through the data of Fig. 2 was transformed to a smooth curve through the data of Fig. 1.

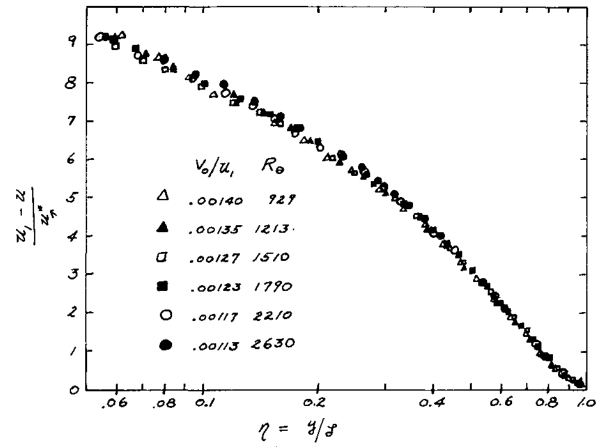

Fig. 1 Experimental confirmation of the U_τ^* form of the velocity defect relation, $dp/dx = 0$, variable v_0.

In Fig. 3, a relation between U_τ^* and U_τ is shown. This relation is based upon the authors' data, previous data,[1, 6] and calculations of U_τ^* made using Eqs. (2) and (3) as discussed previously. For $dp/dx = 0$, the ratio of U_τ^*/U_τ is determined by local conditions at the wall with v_0U_1/U_τ^2 as the governing parameter. From this result it follows that a defect law of the form

$$(U_1 - U)/U_\tau = \phi(y/\delta) \qquad (4)$$

will be followed when v_0U_1/U_τ^2 is constant, because then U_τ^*/U_τ will be constant.

It is concluded that, for transpired turbulent boundary layers with $dp/dx = 0$ and v_0 varying smoothly with x, the

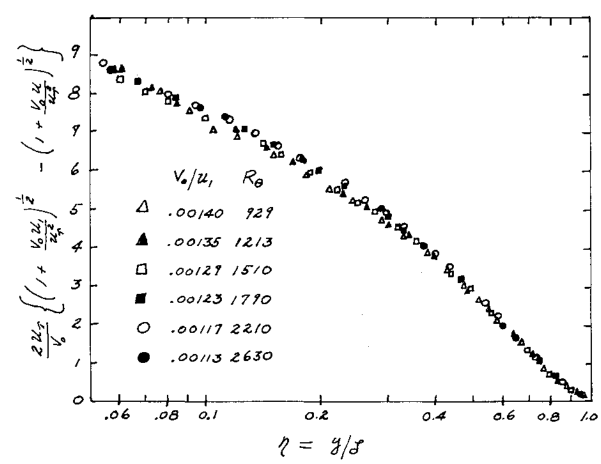

Fig. 2 Experimental confirmation of Stevenson's form of the velocity defect relation, $dp/dx = 0$, variable v_0.

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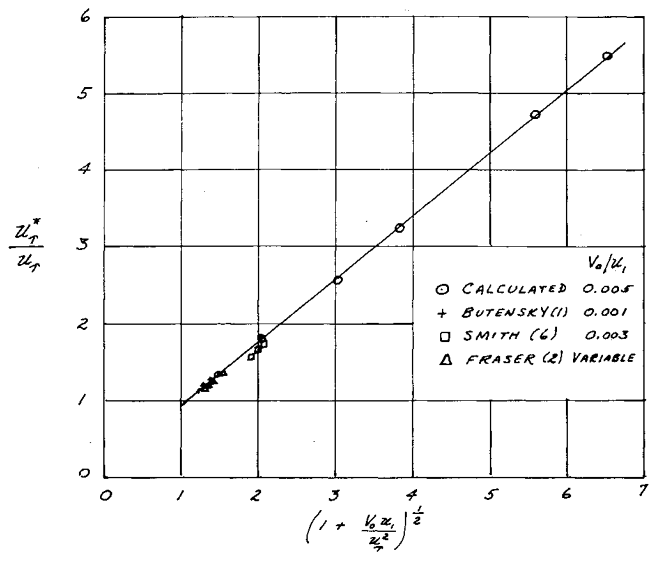

Fig. 3 Relation between U_τ^* and U_τ for transpiration, $dp/dx = 0$.

defect laws represented by Eqs. (1) and (3) are both valid. However, as Figs. 1 and 2 demonstrate, the two correlations are not identical: at the same value of y/δ, the ordinate of Fig. 1 is about 6% greater than the ordinate of Fig. 2. Although the data are subject to uncertainties of this magnitude, the difference between the two correlations is believed significant. The theory behind Eq. (1) is that, for $dp/dx = 0$, the outer portions of transpired and untranspired boundary layers have the same velocity defect law if the shear stress at the inner edge of the outer portion is used to form the scale friction velocity. This shear stress is approximated by the maximum shear stress for positive values of v_0 but is somewhat less than the maximum for the untranspired layer. Data taken in the authors' laboratory for $dp/dx = v_0 = 0$ fall on Fig. 2. If U_τ^* is taken equal to U_τ, the data fall approximately 6% below Fig. 1. However, if U_τ^* is evaluated from the local shear stress at $y/\delta = 0.1$, the data follow Fig. 1.

Stevenson's approach is a major contribution, and for practical calculations is simpler than Eq. (1). However, the implications of Eq. (1) are significant. It is additional evidence of the validity of Clauser's concept that the outer flow of a turbulent boundary layer is a relatively simple region that rides on top of a variable "viscosity" substrate.

References

[1] Butensky, M. S., "The transpired turbulent boundary layer on a flat plate," Sc.D. Thesis, Chemical Engineering Dept., Massachusetts Institute of Technology (1962).

[2] Fraser, M. D., "A study of the equilibrium transpired turbulent boundary layer on a flat plate," Sc.D. Thesis, Chemical Engineering Dept., Massachusetts Institute of Technology (1964).

[3] Mickley, H. S. and Davis, R. S., "Momentum transfer for flow over a flat plate with blowing," NACA TN 4017 (1957).

[4] Mickley, H. S. and Smith, K. A., "Velocity defect law for a transpired turbulent boundary layer," AIAA J. 1, 1685–1687 (1963).

[5] Mickley, H. S., Smith, K. A., and Fraser, M. D., "Velocity defect law for a transpired turbulent boundary layer," AIAA J. 2, 173–174 (1964).

[6] Smith, K. A., "The transpired turbulent boundary layer," Sc.D. Thesis, Chemical Engineering Dept., Massachusetts Institute of Technology (1962).

[7] Stevenson, T. N., "A law of the wall for turbulent boundary layers with suction or injection," College of Aeronautics, Cranfield, England, CoA Aeronautics Rept. 166 (1963).

[8] Stevenson, T. N., "A modified velocity defect law for turbulent boundary layers with injection," College of Aeronautics, Cranfield, England, CoA Aeronautics Rept. 170 (1963).

Drag Coefficients of Particles in Gas-Particle Flow

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THE use of metallized propellents in rocket engines has resulted in the presence of condensed metal oxides in the form of fine dust particles in the exhaust gas. Since the flow in the nozzle is continuously accelerating, and the exchange of momentum and energy between the gas and solid phases take place in finite time, nonequilibrium phenomena set in. To account for these nonequilibrium phenomena, either in a shock wave[2,3] or in a nozzle,[4] the drag and heat-transfer coefficients of the particles must be known. There has been some evidence[3] that, in a flow past a normal shock, the effect of the uncertainties in the heat-transfer coefficient is not as important as those produced by the drag coefficient.

There have been many measurements of the particle drag coefficients.[1,5] The most recent one was carried out by Rudinger.[5] He found that, for 29 μ particles,

$$C_D = 6000\ Re^{-1.7} \qquad 50 < Re < 300$$

The measurement was carried out in a shock tube; the motion of the particles induced by passage of the shock was recorded by streak photography. This drag coefficient has a much steeper slope with respect to the Reynolds number than the drag coefficients used heretofore. These include the Stokes' formula ($24Re^{-1}$), the data obtained for a steady flow past a sphere ($0.48 + 28\ Re^{-0.85}$), and that measured by Ingebo[6] ($27Re^{-8.4}$). An independent measurement using a method different from any of these cited in the foregoing seems to be desirable.

In this note, we shall propose a steady-state method for the measurement of particle drag coefficient in a supersonic flow of gas-particle stream. Essentially, this method consists of impinging a supersonic stream of gas-particle flow onto a wedge. The relaxation process behind the oblique shock wave will enable us to determine the drag coefficient of the particles. Morgenthaler[7] has used this method to determine the characteristics of the gas-particle flow exhausting from a nozzle.

If we neglect the partial pressure of the solid phase, the equation of motion of the particle phase is

$$\lambda(Dq_s/Dt) = \tfrac{1}{2}C_D\pi r_s^2\rho(\mathbf{q} - \mathbf{q}_s)\,|\mathbf{q} - \mathbf{q}_s| \qquad (1)$$

in which λ is the mass of the particles, $\mathbf{q}$ is the velocity vector (subscript s refers to the solid phase), C_D is the drag coefficient, ρ is the density of the gas phase, and r_s is the radius of the solid particles.

If we make the reasonable assumption that the particles do not relax when traversing through the shock wave proper, then the initial relative velocity between the two phases right after the shock is normal to the shock wave. Since the force acting between the two phases is in the direction of the relative velocity, then the deceleration of the particle phase and the acceleration of the gas phase will take place in the direction normal to this shock wave. If we take a coordinate system for which x is normal and y is parallel to the shock wave and write the x-component of velocity u, then Eq. (1) becomes

$$\lambda u_s(du_s/dx) = -\tfrac{1}{2}\pi r_s^2\rho C_D(u - u_s)^2 \qquad (2)$$

By rearrangement,

$$C_D = -\frac{8}{3}\frac{\rho_s}{\rho}\frac{r_s}{[(u_s/u)^2 - 1]}\frac{1}{u_s}\frac{du_s}{dx} \qquad (3)$$

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